Estimating Sediment Transport
Peter Wilcock
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• More background
• Critical and reference shear stresses
• Why it’s hard to estimate transport rates
• Empirical vs formula estimates
• A calibrated method for estimating transport rates in gravel-bed rivers
The first transport problem: incipient motion

The transport model is a defined value of critical Shields Number $\tau_C$

For a gravel-bed river, a reasonable choice of $D$ is the median size of the gravel portion of the bed (the framework), measured with a pebble count. For clean, loose gravel,

$$\tau^* = \frac{\tau_C}{(s-1)\rho g D} \approx 0.03$$

For a gravel bed that has not been entrained for some time, the grains can become weakly cemented, and they can also become arranged into subtle structures that increase their resistance to movement. This can more than double $\tau_c^*$. 

Church, Hassan, Wolcott *Water Res Res* 1998
For smaller sizes, the model is Shields Curve (at least for unisize sediment)

\[ \tau^* = \frac{\tau_c}{(s - 1) \rho g D} \approx 0.03 \]
Uncertainty Exercise

For a simple, wide, prismatic channel, find $Q_c$ directly
What if you are not too sure about some of the values needed to determine $Q_c$?

Like $n$, $D$, and $\tau^*_c$—what do you do?

$$Q = \frac{BhU}{n}$$

$$B = aQ^b$$

$$h = \tau / \rho gS$$

$$U = \frac{\sqrt{S}}{n} h^{2/3} \text{ so}$$

$$Q = \left( aQ^b \right) \left( \frac{\tau}{\rho gS} \right)^{5/3} \frac{\sqrt{S}}{n} \text{ or}$$

$$Q = \left[ \frac{a\sqrt{S}}{n} \left( \frac{\tau}{\rho gS} \right)^{5/3} \right]^{1/1-b}$$

$$\tau_c = \tau^*_c (s-1) \rho gD \text{ so}$$

$$Q_c = \left[ \frac{a}{nS^{7/6}} \left( \frac{\tau^*_c (s-1) D}{nS^{7/6}} \right)^{5/3} \right]^{1/1-b}$$
Suppose your best estimate of Manning’s $n$ is 0.035 and that you are pretty sure that the real value falls between 0.03 and 0.04.

We could approximate your assessment of the value of $n$ with a normal distribution with mean $\bar{n} = 0.035$ & standard deviation $\sigma_n = 0.0025$.

95% of this distribution falls between 0.03 and 0.04, as can be seen in the cumulative frequency plot, so we are saying that the real value of $n$ is 95% likely to fall between 0.03 and 0.04 and that it is more likely to be around the center of the distribution (0.035) than in the tails. We use this distribution to pick values of $n$ in our Monte Carlo simulation.

How does that work? We use a random number generator to pick a number between 0 and 1 and then use this number to find a value of $n$ for the cumulative frequency distribution. For example, for 0.88, $n = 0.0379$ for 0.23, $n = 0.0332$
The Monte Carlo simulation

1. Pick values of $n$, $\tau_c^*$, and $D$ from specified frequency distributions.
2. Calculate critical discharge and transport rate.
3. Repeat 1000 times.
4. Distribution of calculated values gives an estimate of the effect of input uncertainty on calculated critical discharge and transport rate.

\[ Q = BhU \]
\[ B = aQ^b \]
\[ h = \frac{\tau}{\rho g S} \]
\[ U = \frac{\sqrt{S}}{n} h^{2/3} \] so
\[ Q = \left( aQ^b \right) \left( \frac{\tau}{\rho g S} \right)^{5/3} \frac{\sqrt{S}}{n} \] or
\[ Q = \left[ \frac{a\sqrt{S}}{n} \left( \frac{\tau}{\rho g S} \right)^{5/3} \right]^{1-b} \]

\[ \tau_c = \tau_c^*(s-1)\rho gD \] so
\[ Q_c = \left[ \frac{a}{n^b} \left( \frac{\tau_c^*(s-1)D}{n^b} \right)^{5/3} \right]^{1-b} \]

=> Monte Carlo
Monte Carlo Simulation of Qc calculation, using 1000 trials

This is the (uncertain) solution to the threshold channel problem

Need a better answer?  

tracer gravels

LEGEND

<table>
<thead>
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<th>St. Deviation</th>
<th>pdf</th>
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<tr>
<td>Manning's $n$</td>
<td>0.03</td>
<td>0.002</td>
<td>normal</td>
</tr>
<tr>
<td>$\tau^c$</td>
<td>0.045</td>
<td>0.0050</td>
<td>normal</td>
</tr>
<tr>
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<td>0.25</td>
<td>normal</td>
</tr>
<tr>
<td>$D$ (mm)</td>
<td>32</td>
<td></td>
<td>lognormal</td>
</tr>
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</table>

Calculate

<table>
<thead>
<tr>
<th>Qc (m^3/s)</th>
<th>Mean</th>
<th>St. Deviation</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.1</td>
<td>3.4</td>
<td>6.7</td>
<td></td>
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</tbody>
</table>
Application of Monte Carlo approach for assessing risk: threshold channel

Given a channel with a design discharge. The channel is intended to be static. You would like to be 90% sure that the grains in the channel bed will not move at the design discharge.

Solution: specify possible range in \( n \), \( D \), and \( \tau^* \), use Monte Carlo to determine the range of possible discharges. Adjust design until 90% of the calculated values of critical discharge are larger than the design discharge.
The second transport problem: transport rate

Most sediment transport problems are defined in terms of total sediment transport rate $Q_s$ and water discharge $Q$.

A relation giving $Q_s$ as a function of $Q$ is a sediment rating curve.

$$Q_s = aQ^b$$

(1)

$Q_s$ tons per day (kg/hr, Mg/day) and $Q$ ft$^3$/s, or cfs (m$^3$/s, or cms)

Example: with a sediment rating curve such as (1) and a record of discharge (e.g. the daily mean value of $Q$ for 25 years), you can calculate the total sediment load (the sediment yield) by using (1) to calculate the tons of sediment transported for each day and adding up all 9131 or so values to get a total sediment yield for 25 years.

Warning: Sediment Rating Curves can shift & May need a much shorter time interval
If a sediment rating curve is what we use in application, isn’t there a general one available?

Will 1,000 cfs in Summit Ck move the same amount of sediment as 1,000 cfs in the Snake R?

One thing to try: \( Q_s = aQ^b \) \( \frac{Q_s}{Q_{sr}} = \left( \frac{Q}{Q_r} \right)^b \)

It's nice that \( a \) cancelled, but for this model to be general, the rate of change of \( Q_s \) with \( Q \) (i.e. the exponent \( b \)) must be the same everywhere. But we will find that \( b \) varies from little more than one to more than ten!.

Basically, we are looking for a flow variable that can be accurately scaled, such that we have a general model.
A flow variable that does scale well (although it not always easy to determine) is the bed shear stress $\tau$. This is used in most general transport models in common use.

What might a general transport model look like? First, let’s get a more formal introduction to the players. The best way to do this is with a dimensional analysis which will produce a list of dimensionless variables that represent the problem.

This helps because
* the number of dimensionless variables is (usually) 3 less than the full list of variables that we think might be important
* the dimensionless variables often have a useful physical meaning
* Any relation between the dimensionless variables should be general, if we have listed all relevant variables at the start.
Our list: \( q_s = f(\tau, h, D, \rho_s, \rho, \mu, g) \)

\[
\begin{align*}
\frac{L^2}{T} & \quad \frac{M}{LT^2}, \quad L, \quad L, \quad \frac{M}{L^3}, \quad \frac{M}{L^3}, \quad \frac{M}{LT'}, \quad \frac{L}{T^2}
\end{align*}
\]

Dimensional analysis gives:

\( q^\ast = f(\tau^\ast, S^\ast, s, D / h) \)

\[
q^\ast = \frac{q_s}{\sqrt{(s - 1) g D^3}}, \quad \tau^\ast = \frac{\tau}{(s - 1) \rho g D}
\]

\[
S^\ast = \frac{\sqrt{(s - 1) g D^3}}{\mu / \rho}
\]

If \( s = \text{const} \)

\( D \ll h \) and \( S^\ast \text{ large} \)

\( q^\ast = f(\tau^\ast) \)
(Flow force on bed)/(bed area): \( \tau_o \)  

The Shields Number

(Grain weight): \((s - 1) \rho g \frac{\pi}{6} D^3\)

Number of grains/area \( \propto 1/D^2 \)

Shields Number: \[ \frac{\text{Flow Force}}{\text{Grain weight}} = \frac{\tau_o}{(s - 1) \rho g D} \equiv \tau^* \]

This is THE most important variable in sediment transport

---

The Einstein Transport Parameter

Volumetric transport rate/width \( q_s \)

Sediment fall velocity \( w \propto \sqrt{(s - 1)gD} \)

\[ q^* = \frac{\text{transport rate}}{\text{fall velocity} \times \text{grain size}} = \frac{q_s}{\sqrt{(s - 1)gD^3}} \]
A typical transport model: \( q^* = a(\tau^* - \tau_c^*)^b \)

\( \tau_c^* = \frac{\tau_c}{(s-1)\rho gD} \)

Meyer-Peter & Müller: \( q^* = 8(\tau^* - \tau_c^*)^{3/2} \)

\[
\frac{q_s}{\sqrt{(s-1)gD^3}} = 8\left(\frac{\tau}{(s-1)\rho gD} - \frac{\tau_c}{(s-1)\rho gD}\right)^{3/2}
\]

\[
q_s = \frac{8}{(s-1)g\rho^{3/2}}(\tau - \tau_c)^{3/2}
\]

\( q_s = 78.7(\tau - \tau_c)^{3/2} \)

for \( q_s \) in (tons/day) and \( \tau, \tau_c \) in (Pa)

“simpleMPM.xls”
A typical transport model: \[ q^* = a(\tau^* - \tau_c^*)^b \]

\[ \tau_c^* = \frac{\tau_c}{(s - 1) \rho g D} \]

Meyer-Peter & Müller: \[ q^* = 8(\tau^* - \tau_c^*)^{3/2} \]

**One more transport variable, \( W^* \)**

\[ W^* = \frac{q^*}{(\tau^*)^{3/2}} = \frac{(s - 1) g q_s}{(\tau / \rho)^{3/2}} \]

(sorry!)

**One more stress, the reference stress \( \tau_r \)**

Convert the M-PM formula to \( W^* \) and incorporate a reference shear stress. First, we divide M-PM by \((\tau^*)^{3/2}\) to get

\[ W^* = 8 \left( 1 - \frac{\tau_c^*}{\tau^*} \right)^{3/2} \]

At \( \tau^* = \tau_r^* \), \( W^* = W_r^* = 0.002 \). Dividing by 8, raising both sides to the 2/3 power produces

\[ 0.004 = 1 - \frac{\tau_c^*}{\tau_r^*} \]

\[ \tau_c^* = 0.996 \tau_r^* \]

Meyer-Peter & Müller: \[ W^* = 8 \left( 1 - 0.996 \frac{\tau_r^*}{\tau} \right)^{3/2} \]
Reference shear stress? Critical shear stress?

Meyer-Peter & Müller

\[ W^* = 8 \left( 1 - \frac{\tau_c^*}{\tau^*} \right)^{3/2} \]

or

\[ W^* = 8 \left( 1 - \frac{0.996 \tau_r^*}{\tau^*} \right)^{3/2} \]
How the reference shear stress gets used

Gravel transport rate $q_{bi}$ [g / (m•s)]

Bed Shear Stress $\tau$ (Pa)

Dimensionless transport rate $W^*$

Bed Shear Stress $\tau$ (Pa)

Dimensionless transport rate $W^*$

Bed Shear Stress $\tau / \tau^*$

J06
J14
J21
J27
BOMC
The Difference Between $\tau_c$ and $\tau_r$

$\tau_c$: boundary between motion & no motion. Definable exactly for an individual grain; definable statistically for a river bed. 

*Hard to measure, tracer grains are your best shot*

$\tau_r$: the stress associated with a small, agreed-upon transport rate ($W^* = 0.002$) provides a threshold for transport estimate.

*Easy to determine from transport observations; new efforts using tracer grains*

Tracers (painted rocks, magnetic rocks, radio rocks, rock scum) answer the *competence* question “did the grain move at all?” (place tracers, return after flow, measure # moved, repeat for range of flows, develop relation between %entrained, grain size, and flow)

Need large numbers of grains for reliable sample

Need to place “naturally”
Fig. 2.113.—Sediment Discharge as Function of Water Discharge for Colorado River at Taylor's Ferry Obtained from Observations and Calculations by Several Formulas

Fig. 2-33. Plot of several bed load functions found in the literature.
Example calculation using Meyer-Peter and Muller for a channel with slope $S = 0.002$, roughness $n = 0.025$, and width $b = 15\text{m}$. The solid curve uses $\tau^*_c = 0.045$ and $D = 45\text{ mm}$. The dashed curve uses $\tau^*_c = 0.045$ and $D = 30\text{ mm}$.

Note that at discharge $Q = 55\text{ cms}$, one curve indicates zero transport and the other a transport rate of 80,000 kg/hr.
Sediment Transport *Why it's hard to estimate I*

Why it's hard to estimate I:

Transport rate is a **steep, nonlinear** function of bed shear stress.

Small error in \( \tau \) can produce big error in \( q_s \).

Three things make it difficult to accurately estimate \( \tau \):

1. Accelerations in **unsteady, nonuniform** (i.e. real) flow
2. Only a portion of the total \( \tau \) acts to transport sediment
3. \( \tau \) varies locally. Because transport is a nonlinear fn of \( \tau \), estimates based on total \( \tau \) will be in error.
Three things make it difficult to accurately estimate $\tau$:

1. Accelerations in unsteady, nonuniform (i.e. real) flow

2. Only a portion of the total $\tau$ acts to transport sediment

3. $\tau$ varies locally. Because transport is a nonlinear function of $\tau$, estimates based on total $\tau$ will be in error

$$\tau_0 = \rho g R \left( S - \frac{\partial h}{\partial x} - \frac{U}{g} \frac{\partial U}{\partial x} - \frac{1}{g} \frac{\partial U}{\partial t} \right)$$

$\tau_0 = \rho g R \left( S - \frac{\partial h}{\partial x} - \frac{U}{g} \frac{\partial U}{\partial x} \right)$

NOTE:

Backwater programs compute $S_f$
The Drag Partition

So far, the stress we are talking about is the total stress \( \tau_0 \), or force per area, that the flow exerts on the boundary of the channel.

Three things make it difficult to accurately estimate \( \tau \):

1. Accelerations in unsteady, nonuniform (i.e. real) flow
2. Only a portion of the total \( \tau \) acts to transport sediment
3. \( \tau \) varies locally. Because transport is a nonlinear fn of \( \tau \), estimates based on total \( \tau \) will be in error

But only a portion of the total \( \tau_0 \) acts on the sediment grains to produce transport. To estimate transport rates, we need to partition \( \tau_0 \) into the part that acts only on the sediment grains. We call this the skin friction, or grain stress \( \tau' \).

There is no direct way to do this. We will develop an approximate method, using Manning’s equation.
Solve the Manning eqn for the boundary stress:

\[ U = \frac{\sqrt{SR^{2/3}}}{n} \]
\[ (\rho g)^{2/3} S^{1/6} nU = (\rho gRS)^{2/3} \]
\[ \rho gS^{1/4} (nU)^{3/2} = \tau_0 \]

Estimate the part of the roughness due to the bed grains:

\[ n_D = 0.013D^{1/6} \rightarrow \rho g (0.013)^{3/2} (SD)^{1/4} U^{3/2} = \tau' \]

Manning-Strickler Roughness for \( D \) in mm

Combine \( \frac{U}{u_\ast} \approx 8.1 \left( \frac{R}{k_s} \right)^{1/6} \)

and \( u_\ast = \sqrt{gRS} \)

to get \( n_D = \frac{k_s^{1/6}}{8.1\sqrt{g}} \)

\[ \tau' = 17 (SD_{65})^{1/4} U^{3/2} \]

where we use \( D = 2D_{65} \)

\[ \rho = 1000 \text{ kg/m}^3 \text{ & } g = 9.81 \text{ m/s}^2 \]
Using both the grain roughness and total roughness, we have an estimate of the portion of the total stress acting on the grains – the grain stress.

\[ \tau_o = \rho g S^{1/4} \left( nU \right)^{3/2} \]

\[ \tau' = \rho g S^{1/4} \left( n_D U \right)^{3/2} \]

\[ \frac{\tau'}{\tau_o} = \left( \frac{n_D}{n} \right)^{3/2} \]

\[ \tau' = \left( \frac{n_D}{n} \right)^{3/2} \tau_o \]

Basically the same drag partition suggested by Meyer-Peter & Muller in the ‘40s. Using Manning’s \( n \) incorporates general experience, supports application in HEC-RAS.
Three things make it difficult to accurately estimate $\tau$:

1. Accelerations in *unsteady, nonuniform* (i.e. real) flow

2. Only a portion of the total $\tau$ acts to transport sediment

3. $\tau$ varies locally. Because transport is a nonlinear function of $\tau$, estimates based on total $\tau$ will be in error

The power of an average does not equal the average of a power!

$$q_s = 78.7 (\tau - \tau_c)^{3/2}$$

for $q_s$ in (tons/day)

and $\tau, \tau_c$ in (Pa)
Sediment Transport *Why its hard to estimate II*

What is $D$ for a reach?

What if $D$ is not even in the reach?

Critical shear stress depends linearly on grain size, but the grain size of the transport is poorly known!

Transport rate is a steep, nonlinear function of bed shear stress

Small error in $\tau_c$ can produce big error in $q_s$
Estimating Sediment Transport: Why it’s hard

- Sediment transport is a local process: between individual grains and the near-bed flow they encounter
- The transport process is highly nonlinear: a very steep function with a threshold between motion and no motion
- Real streams have considerable variability in topography, flow, and bed composition
- The local flow acting on grains is hard to estimate
- The distribution of bed grain sizes is hard to determine
- You never know either local flow or local bed composition in detail, so you have to estimate
- Errors in estimated transport rates are typically very large (often well in excess of an order of magnitude!)
- Next, we examine an approach to provide reliable transport estimates, of greater accuracy, with acceptable effort
Estimating Bed-Material Transport in Gravel-Bed Rivers
Dealing with Grain Size

- Bed/Suspended Load vs Wash/Bed Material Load
- How many sizes?
Dealing with Grain Size

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Mode</th>
<th>Source</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Washload</td>
<td>suspension</td>
<td>?? uplands, banks, backwaters, ...</td>
<td>true washload: (a) Transport not predictable (b) too little in bed to affect transport of other fractions</td>
</tr>
<tr>
<td>Clay, silt, fine sand</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bed Material – Fine</td>
<td>bed load or</td>
<td>interstices, stripes and dunes, subsurface</td>
<td>grain path in near-bed region dominated by larger grains; hard to sample &amp; model</td>
</tr>
<tr>
<td>med-crs sand, pea gravel</td>
<td>suspension</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bed Material – Coarse</td>
<td>bed load</td>
<td>bed framework</td>
<td>displacements generally rare and hard to capture</td>
</tr>
<tr>
<td>med-crs gravel, cobble</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bed Material – Huge</td>
<td>immobile at</td>
<td>bed</td>
<td>Requires decision regarding grains to exclude from the transportable population</td>
</tr>
<tr>
<td>Boulder</td>
<td>typical high flows</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We focus on fine and coarse bed material
What about more fractions?

Many-fraction models available: including ones based on the grain size of the bed surface, which allow for the prediction of transient conditions.

These models are fragile – output is sensitive to the quality of the input.

To simulate transport at a particular location, at a particular time,

need large amounts of detailed bed and flow data

Use these models to ask more general questions:

e.g. change in bed composition & transport with a change in flow releases from dams or a change in sediment supply from dam removal

We will apply such a model later …
Estimating Sediment Transport: Three overarching constraints bound the problem

- Large spatial & temporal **variability**
  
  Need BIG samples

- **Strongly nonlinear** processes
  
  bed-material transport is a local affair
  
  Modeling with spatial/temporal averages produces large error

- **Sparse information** available relative to that needed to model the transport
  
  Modeling with high spatial/temporal resolution requires vast amounts of data

- Generally, we need a **robust** approach
1. Estimating Transport: Formulas

- Threshold channels, Incipient motion: What is $Q_c$?
- Mobile-bed channels, Transport rate: What is $Q_s$?

We have trouble estimating $\tau_c$, as well as hydraulic parameters.
Calculating cumulative sediment transport in a simple, prismatic channel, over a hydrograph

Major uncertainty in \( n, D, \) and \( \tau_c^* \) – what are the consequences?

Replace \( \tau \) in Meyer-Peter & Muller:

\[
Q_s = 8B_o \left( \frac{2650}{1000} \right) (3600) \sqrt{(s-1)gD}^3 \left( \frac{nQ^{1-b}}{a} \right)^{3/5} \frac{S^{0.7}}{(s-1)D - \tau_c^*} \]

\[
h = \frac{Q}{BU}
\]

\[
B = aQ^b
\]

\[
U = \frac{\sqrt{S}}{n} h^{2/3}
\]

\[
h = \frac{nQ}{\left( aQ^b \right) \sqrt{S} h^{2/3}}
\]

\[
h = \left( \frac{nQ^{1-b}}{a \sqrt{S}} \right)^{3/5}
\]

Replace \( h \) in

\[
\tau = \rho ghS
\]

\[
\tau = \rho g \left( \frac{nQ^{1-b}}{a} \right) S^{0.7}
\]
Monte Carlo Simulation of $\Sigma Qs$ calculation, using 1000 trials

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<tr>
<th>Mean</th>
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<th>Interval</th>
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</thead>
<tbody>
<tr>
<td>14,997</td>
<td>2,806</td>
<td>5,509</td>
</tr>
</tbody>
</table>

“MonteCarloTransport.xls”

Useful for evaluating uncertainty in sediment balance between sections
• If a good transport estimate is required: field observations are needed (no different than Manning’s eqn.)

• Trying different equations to evaluate uncertainty by using different transport formulas misses the point: the main source of uncertainty is in the input!
2. Estimating Transport: Sampling

Option 1: trap all the transport in a weir, slot, or pool

Option 2: point samples: portable or pit/net-frame samplers installed on the bed.

Option 1 is best, but generally not practical

Option 2 is ≈ practical, but involves larger error, some risk, some luck, and lots of effort
Sampling I

Transport field highly variable in space & time

⇒ Need LARGE samples!

Define *Sampling Intensity* for a 6.5m stream

<table>
<thead>
<tr>
<th>Total Transport</th>
<th>[6.5m] X [3600s] = <strong>23,400</strong> m•s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hand-held Sampling Transect</td>
<td>[13 * 0.075m] X [120s] = <strong>117</strong> m•s (0.5%)</td>
</tr>
<tr>
<td>Pit/Trap Sampler</td>
<td>[5 * 0.305 m] X [3600s] = <strong>5,490</strong> m•s (23.5%)</td>
</tr>
</tbody>
</table>

*So, pit or net-frame traps look pretty good …*
Pit samplers fill up at high transport rates

Photo J Pizzuto, U Delaware, home of big samples

So do net-frame samplers
<table>
<thead>
<tr>
<th>Transport Fraction</th>
<th>Physical Sampler</th>
<th>Surrogate Sampler</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wash load</td>
<td>Std. suspended sediment samplers</td>
<td>Various sound &amp; light</td>
</tr>
<tr>
<td>Fine Bed Material</td>
<td>Helley-Smith, w/ base or suction?</td>
<td></td>
</tr>
<tr>
<td>Coarse Bed Material</td>
<td>Net or pit traps, or ponds</td>
<td>Acoustic?</td>
</tr>
</tbody>
</table>
3. Estimating Sediment Transport

• **Direct sampling**
  + gives a direct relation between $Q$ and $Q_s$
  - requires a big effort
  - cannot predict future conditions
  - error: is it random or systematic?
  - **Handheld samplers**: Scooping, perching, limited grain size & TINY SAMPLES
  - **Pit/Net-frame samples**: better sampling, only at low rates

• **Formula predictions**
  + can predict future changes
  - highly inaccurate
    - flow hard to scale
    - boundary conditions poorly known
The alternative? Join forces.
Need for both accuracy and efficiency indicate that the future is a combination of **simple robust models** and **efficient measurement**.

A first cut, using today’s technology

- If pit/trap samplers can collect good samples of small transport rates, why not use these to calibrate a model of coarse bed-material transport
- **GOOD** samples! (long duration, spatially extensive)
- Two sizes: sand and gravel
- Combine in robust framework that is insensitive to major sources of error
Transport Formula (Gravel)

\[
W_i^* = \begin{cases} 
11.2 \left( 1 - 0.846 \frac{\tau_r}{\tau} \right)^{4.5} & \tau > \tau_r \\
0.0025 \left( \frac{\tau}{\tau_r} \right)^{14.2} & \tau < \tau_r 
\end{cases}
\]

Calibrate \( \tau_r \)

Dimensionless transport rate

\[
W^* = \frac{q^*}{\tau_r^{3/2}} = \frac{(s - 1)gq_b}{(\tau / \rho)^{3/2}}
\]

Choice of formula does not make that much difference!

Formula provides the trend, but the samples provide the accuracy.
Transport formula based on shear stress $\tau$, so we need a drag partition relation to get grain stress from discharge. We return to our Manning-Strickler formula:

$$\tau = 17(SD_{65})^{1/4}U^{3/2}$$

- For Cub River:
  - Slope $S = 0.02$
  - $D_{65} = 90$ mm
  - Mean velocity $U = 0.46 Q^{0.42}$
Observed Transport Function

\[ W^* \]

\[ \frac{\tau}{\tau_r} \]

\[ Q_s (\text{m}^3/\text{s}) \]

\[ \tau_c = 0.039 \]
SRC.xls

This worksheet for GRAVEL transport

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
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<td>3</td>
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<td>4</td>
<td>D65</td>
<td>90 (mm)</td>
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<td>D50</td>
<td>42 (mm)</td>
<td>D50 of bed subsurface</td>
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<td>6</td>
<td>fg</td>
<td>0.50</td>
<td>proportion travel in bed</td>
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<td>7</td>
<td>b</td>
<td>15.0 (m)</td>
<td>width of bed material, not channel</td>
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<td>(Pa) Adjust the reference shear stress</td>
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<td>Reference Shields Number; is it reasonable?</td>
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<td>Qsg</td>
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<td>(m^3/s)</td>
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</table>

Enter parameters of velocity rating curve here

k = 0.63  m = 0.34  Qc = 0.00

\[ U = k(Q - Q_t)^m \]

The cells below contain values of k and m fitted to the (Q, U) values for specified Qc

<table>
<thead>
<tr>
<th>Q</th>
<th>U</th>
<th>Q-Qt</th>
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<tbody>
<tr>
<td>(m^3/s)</td>
<td>(m/s)</td>
<td>(m^3/s)</td>
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<tr>
<td>1.568</td>
<td>0.73</td>
<td>1.568</td>
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<tr>
<td>4.225</td>
<td>1.04</td>
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<td>5.946</td>
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<tr>
<td>12.340</td>
<td>1.47</td>
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<tr>
<td>13.326</td>
<td>1.50</td>
<td>13.326</td>
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</table>

0.3382  -0.1983 LINEST results
Transport samples provide accuracy by

(1) Establishing grain size $D$ of the transport &

(2) Scaling the flow

Calculate grain stress: $\tau = 17 (SD_{65})^{0.25} U^{1.5}$

for two different flows & take the ratio: $\frac{\tau_1}{\tau_2} = \left(\frac{U_1}{U_2}\right)^{1.5}$

Suppose $U = aQ^b$

then $\frac{\tau_1}{\tau_2} = \left(\frac{Q_1}{Q_2}\right)^{1.5b}$

If you calibrate, the transport formula is needed only to give the change in transport with the change in discharge.
Most of the transport occurs at high flows & you base your estimate on samples at low flow?

Estimating bed-material transport in gravel-bed rivers

- **Conceptual basis**
  - fine and coarse bed material
  - (supply of one affects the transport of the other)
- **Sampling**
  - standard needed for minimum sample size
  - fine bed material – Helley-Smith or ?
  - coarse bed material – pit or net frame samplers
  - big rivers – ???
- **Modeling**
  - 2-fraction model captures essential interaction between fine & coarse
  & facilitates integral measure of reach grain size
  - But it can’t do everything (armoring; change in sand or gravel size)
- **Future**
  - combine simple, robust models with efficient monitoring
  - can be done now in wadeable streams, although effort is non-trivial
SRC & BAGS
Software and accompanying manual to support estimates of bed-material transport rates in gravel-bed rivers. Stream Systems Technology Center, (the “Stream Team”), US Forest Service

BAGS (Yantao Cui, Stillwater Science): supports variety of transport formulas, allows variety of input, includes calibrated approach

User’s Manual by John Pitlick
Primer by Peter Wilcock

TOWARD A PRACTICAL METHOD FOR ESTIMATING SEDIMENT-TRANSPORT RATES IN GRAVEL-BED RIVERS

PETER R. WILCOCK*
Department of Geography and Environmental Engineering, Johns Hopkins University, Baltimore, MD 21218, USA
Your assignment – calculate transport for Cub R

Apply calculated sediment transport rates to estimates of annual load and effective discharge