Hydraulics Overview

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- General observation & some terms
- Conservation of mass & force balance
- Flow resistance – what sets the depth?
- Using all the tools at once
- Energy in open channel flow
  - Rapidly varied & gradually varied flow
- Water surface, or backwater modeling
The open channel toolbox
(for flow and sediment transport)

- Conservation Relations
  Water Mass (aka continuity)
  Momentum (aka $F = ma$, Newton’s 2nd Law)
  Energy

- Constitutive Relations
  Flow resistance (we’ll use Manning eq.)
  Sediment transport (Wednesday)

- Morphodynamics — add conservation of sediment mass
  (Wednesday/Thursday)
• Distorted view
  (channels are much wider than they are deep;
  \( b/h = 20 \) is a narrow channel)

• Width \( b \), area \( A \), wetted perimeter \( P \)
• Hydraulic radius \( R = A/P \)
• Mean depth \( h = A/b \)
• Often, \( b \approx P \), so \( R \approx h \)
  e.g. for a rectangular channel

\[
R = \frac{bh}{b + 2h} \approx \frac{bh}{b} = h
\]

STEADY flow: not changing in time
\[
\frac{\partial}{\partial t} = 0
\]

UNIFORM flow: not changing in space
\[
\frac{\partial}{\partial x} = 0
\]

NORMAL flow: steady and uniform (!)
Continuity

• Conservation of water mass
• Just accounting, like your checkbook
• Rate of change of water mass in reach = net rate of input and output
• Inputs and outputs:
• Constant discharge: input = output, no change in water mass in reach

\[ Q_1 = U_1 A_1 \quad \text{and} \quad Q_2 = U_2 A_2 \]

For \( Q_1 = Q_2 \), \( \frac{U_2}{U_1} = \frac{A_1}{A_2} \)

\( Q \) is water discharge (e.g. ft\(^3\)/s or m\(^3\)/s); \( U \) is mean velocity (e.g. ft/s or m/s)
Continuity for unsteady and nonuniform 1d flow —
Now depth $h$ varies in both time and space

$$\Delta S = I - 0$$

For a control volume, the rate of change of mass = the net rate of mass flowing through the sides

$$\frac{\partial \rho \nabla}{\partial t} = \rho U h B|_x - \rho U h B|_{x + \Delta x} = -\frac{\partial (\rho U h B)}{\partial x} \Delta x$$

For constant $\rho, B$. $$\frac{\partial h \Delta x}{\partial t} = -\frac{\partial (U h)}{\partial x} \Delta x$$

or $$\frac{\partial h}{\partial t} = -\frac{\partial (U h)}{\partial x}$$

$$\frac{\partial h}{\partial t} + \frac{\partial (U h)}{\partial x} = 0$$

If $\frac{\partial}{\partial t} = 0$, we recover $U h B = Q = \text{constant}$
Volume of water: $AL$
Weight of water: $\rho gAL$
Downslope component of weight of water: $\rho gALS$

$\tau_o$ is the boundary shear stress - the flow force per unit area - it drives the sediment transport.

Boundary stress: $\tau_o$
(stresss is force/area)

Boundary force: $\tau_o PL$

$\tau_o PL = \rho gALS$

$\tau_o = \rho gRS$

The ‘depth-slope’ product $\tau_o \approx hS$  ($h$ in cm, $S$ in %, $\tau_o$ in Pa)
1b. Boundary shear stress — Unsteady, Nonuniform Flow

We still use Newton’s second law, \[ \Sigma F = ma \]

But the flow is accelerating in (1) space and (2) time. These accelerations are balanced by the sum of forces, to which we now must add a third force, due to the difference in flow depth down the reach.
If the flow is predominantly one-dimensional (no rapid changes in width or depth that give rise to vertical or lateral accelerations), we get the 1d St. Venant Eqn:

$$ma = F$$

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} = gS - g \frac{\partial h}{\partial x} - \frac{\tau_o}{\rho R}$$

"local" acceleration  "convective" acceleration  body force  pressure force  boundary force

Rearrange to solve for $\tau_o$:

$$\frac{\tau_0}{\rho g R} = S_o - \frac{\partial h}{\partial x} - \frac{U \partial U}{g \partial x} - \frac{1}{g \partial t} \frac{\partial U}{\partial t}$$

Steady, uniform flow

Steady, nonuniform flow

Unsteady, nonuniform flow

An approximate derivation is in the primer.
We can rearrange the St. Venant eqn, to solve for shear stress

\[ \tau_0 = \rho g R \left( S - \frac{\partial h}{\partial x} - \frac{U}{g} \frac{\partial U}{\partial x} - \frac{1}{g} \frac{\partial U}{\partial t} \right) \]

1. For steady, uniform flow, we recover the depth-slope product

2. But flow is never perfectly steady and nonuniform!

3. So, how do we know whether the other terms are important or not?

4. Are we making an assumption or an approximation?

NOTE: \[ \tau_0 = \rho g R \left( S - \frac{\partial h}{\partial x} - \frac{U}{g} \frac{\partial U}{\partial x} \right) = \rho g R S_f \]

Backwater programs (HEC-RAS) compute:

\[ S_f = \frac{d}{dx} \left( z_b + h + \frac{U^2}{2g} \right) \]

→ flood.xls
Flow from a tributary at point a increases rapidly from \( Q_1 \) to \( Q_2 \) over \( \Delta t \). At time \( \Delta t \), any increase in flow has not yet reached a distance \( \Delta x \) downstream, creating a nonuniform flow ‘wedge’. The flow is unsteady (it is increasing with time) and it is nonuniform (it is increasing upstream).

**Question:** do the unsteady and nonuniform terms contribute significantly to \( \tau_o \)?

Given a rectangular channel:

<table>
<thead>
<tr>
<th>Channel width ( b )</th>
<th>15 (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel slope ( S_0 )</td>
<td>0.001</td>
</tr>
<tr>
<td>Hydraulic roughness ( n )</td>
<td>0.035</td>
</tr>
<tr>
<td>( Q_1 )</td>
<td>10 (m(^3)/s)</td>
</tr>
<tr>
<td>( Q_2 )</td>
<td>15 (m(^3)/s)</td>
</tr>
<tr>
<td>( \Delta t )</td>
<td>3600 (s)</td>
</tr>
</tbody>
</table>

**Enter Data only in green cells**

- Flow is unsteady and nonuniform, so we ask: which terms in the St Venant Eqn are significant?
- How does the shear stress change, approximately, over this reach of nonuniform flow?

>>> we need to estimate the magnitude of the different terms in the St Venant Eqn.

The length of the flood wave \( \Delta x \) is found from \( U_f = \frac{\Delta x}{\Delta t} \), where \( U_f \) is the speed of the flood wave and is approximately 1.5 times mean velocity in the reach.

\( \Delta x = 4470 \) (m)

We need to determine depth and velocity at the upstream and downstream end of the reach.

<table>
<thead>
<tr>
<th>UPSTREAM</th>
<th>DOWNSTREAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_u )</td>
<td>1.12 (m)</td>
</tr>
<tr>
<td>( h_d )</td>
<td>0.87 (m)</td>
</tr>
<tr>
<td>( U_u )</td>
<td>0.89 (m/s)</td>
</tr>
<tr>
<td>( U_d )</td>
<td>0.77 (m/s)</td>
</tr>
</tbody>
</table>

We also need the mean velocity and hydraulic radius for the reach.

| \( R \) | 0.88 (m) |
| \( U \) | 0.83 (m/s) |
| \( \Delta h \) | -0.253 (m) |
| \( \Delta U \) | -0.12 (m/s) |

Now we can compute all of the components in the brackets of the St Venant eqn.

\[
\tau_0 = \rho g R \left[ S_0 - \frac{d h}{d x} - \frac{U \Delta U}{g} - \frac{1}{g} \frac{1 \Delta U}{\Delta t} \right]
\]

To make approximation, express in difference form:

\[
\tau_0 = \rho g R \left[ S_0 - \frac{\Delta h}{\Delta x} - \frac{U \Delta U}{g} - \frac{1}{g} \frac{1 \Delta U}{\Delta t} \right]
\]

We drop if any of these terms are more than an order of magnitude smaller.

Mean \( \tau_o \) in nonuniform reach, approximated from St Venant eqn.

\( \tau_o = 9.16 \) Pa

\( \tau_o \) in steady uniform flow before flood

\( \tau_o = 7.65 \) Pa
Use St. Venant to consider sediment transport for a steady river flow entering a reservoir. Assume a wide channel, so $R = h$. As flow enters the reservoir, it deepens and slows (by continuity!). Which effect wins out?

$$\tau_0 = \rho g R \left( S - \frac{\partial h}{\partial x} - \frac{U}{g} \frac{\partial U}{\partial x} - \frac{1}{g} \frac{\partial U}{\partial t} \right)$$

Consider $\left( -\frac{\partial h}{\partial x} - \frac{U}{g} \frac{\partial U}{\partial x} \right)$ for the case of constant width $b$ (for simplicity)

such that $q = U h = \text{const}$ and $\frac{\partial q}{\partial x} = 0 = \frac{\partial (U h)}{\partial x} = U \frac{\partial h}{\partial x} + h \frac{\partial U}{\partial x}$ such that

$$\frac{\partial U}{\partial x} = - \frac{U}{h} \frac{\partial h}{\partial x}. \quad \text{Replace } \frac{\partial U}{\partial x} \text{ in the first equation above, giving}$$

$$\left( -\frac{\partial h}{\partial x} + \frac{U}{g} \frac{\partial h}{\partial x} \right) = \frac{\partial h}{\partial x} \left( \frac{U^2}{gh} - 1 \right). \quad \text{Because } \frac{U^2}{gh} < 1 \text{ in a reservoir,}$$

the entire quantity $\left( -\frac{\partial h}{\partial x} - \frac{U}{g} \frac{\partial U}{\partial x} \right)$ is negative. Because $\frac{\partial h}{\partial x}$ is itself positive

for flow entering a reservoir, we see that $\tau_o$ is reduced from $\rho gh S$.

A smaller value of $\tau_o$ means that transport capacity decreases and sediment deposits.

Does boundary stress $\tau_o$ increase or decrease as we enter the reservoir?
Consider a typical problem:
Specify a channel, with known geometry
(fluid $\rho$ and acceleration of gravity $g$ are also known).
For a specified discharge $Q$, what is the flow depth $h$ for steady uniform flow?
Continuity gives us $U$ (when $A(h)$ is known).
Momentum gives $\tau_o$ (when $R(h)$ is known).
Energy gives $S_f = S_o$ for steady uniform flow.
We need one more relation to find depth $h$!
This is a flow resistance relation (a constitutive relation).
Flow Resistance

- A relation between velocity, flow depth, boundary stress, and boundary roughness
- We’ll use Manning’s eqn.
- Prefer an eqn. with constant roughness ($n$)
- Using continuity:

\[ U = \frac{\sqrt{S}}{n} R^{2/3} \]

\[ Q = UA = \frac{\sqrt{S}}{n} AR^{2/3} \]

*S is the slope of the ‘energy grade line’, which is also the slope of the channel for uniform flow*
• For a useful approximation, consider a wide \((R \approx h)\) rectangular channel
• Solve for depth
• In field, relation between \(Q, A, \& R\) more complex, though not more complicated
• In practice, a flow resistance relation is often used to find the flow depth for a specified flow in a given channel

\[ Q = \frac{\sqrt{S}}{n} AR^{2/3} \]

\[ Q = \frac{\sqrt{S}}{n} (bh) h^{2/3} \]

\[ Q = \frac{\sqrt{S}}{n} bh^{5/3} \]

\[ h = \left(\frac{nQ}{b\sqrt{S}}\right)^{3/5} \]

\(\text{Manning’s Eq., 3 ways:}\)

\[ n = \frac{b\sqrt{S}h^{5/3}}{Q} \]

\[ Q = \frac{b\sqrt{S}h^{5/3}}{n} \]

\[ h = \left(\frac{nQ}{b\sqrt{S}}\right)^{3/5} \]
The hard part: determining roughness ‘n’

• How to measure:
  survey cross section and reach, which gives S and A, R as a function of water surface elevation (aka stage or WSEL)
• Measure discharge at a known stage, solve for \( n \)

\[
Q = \frac{\sqrt{S}}{n} AR^{2/3}
\]
\[
n = \frac{\sqrt{S}}{Q} R^{2/3} A
\]

• If you don’t measure, you are guessing
• Picture books, tables
• Formulas \( n = fn(D, R, \text{veg.}, ...) \)
Roughness Characteristics of Natural Channels

By HARRY H. BARNES, Jr.

U.S. GEOLOGICAL SURVEY WATER-SUPPLY PAPER 1849

Color photographs and descriptive data for 50 stream channels for which roughness coefficients have been determined

USGS
science for a changing world

10–1550. Provo River near Hailstone, Utah

Gage location.—Lat 40°36', long 111°22', in SE 1/4 sec. 34, T. 2 S., R. 5 E., on right bank 3 miles upstream from Ross Creek and Hailstone. Section 1 is about 120 ft upstream from gage.

Drainage area.—233 sq mi.

Date of flood.—June 13, Oct. 7, 1952.

Gage height.—4.66 ft, 1.58 at gage; 5.66 ft, 2.14 ft at section 1.

Peak discharge.—1,200 cfs, 64.8 cfs.

Computed roughness coefficient.—Manning n = 0.045; 0.073.

Description of channel.—Bed and banks consist of smooth rounded rocks as much as 1 ft in diameter. Some undergrowth is below water elevations of June 13.

Reach properties

<table>
<thead>
<tr>
<th>Section</th>
<th>Area (sq ft)</th>
<th>Top width (ft)</th>
<th>Mean depth (ft)</th>
<th>Hydraulic radius (ft)</th>
<th>Mean velocity (ft per sec)</th>
<th>Length (ft) between sections</th>
<th>Fall (ft) between sections</th>
</tr>
</thead>
<tbody>
<tr>
<td>June 13, 1952</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.</td>
<td>184</td>
<td>47</td>
<td>3.9</td>
<td>3.70</td>
<td>6.52</td>
<td>88</td>
<td>0.67</td>
</tr>
<tr>
<td>2.</td>
<td>171</td>
<td>49</td>
<td>3.5</td>
<td>3.33</td>
<td>7.02</td>
<td>109</td>
<td>1.04</td>
</tr>
<tr>
<td>3.</td>
<td>173</td>
<td>55</td>
<td>3.1</td>
<td>3.02</td>
<td>6.95</td>
<td>117</td>
<td>1.10</td>
</tr>
<tr>
<td>4.</td>
<td>175</td>
<td>48</td>
<td>3.6</td>
<td>3.43</td>
<td>6.95</td>
<td>117</td>
<td>1.10</td>
</tr>
<tr>
<td>5.</td>
<td>183</td>
<td>55</td>
<td>3.3</td>
<td>3.22</td>
<td>6.56</td>
<td>116</td>
<td>1.04</td>
</tr>
</tbody>
</table>

Oct. 7, 1952

<table>
<thead>
<tr>
<th>Section</th>
<th>Area (sq ft)</th>
<th>Top width (ft)</th>
<th>Mean depth (ft)</th>
<th>Hydraulic radius (ft)</th>
<th>Mean velocity (ft per sec)</th>
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<th>Fall (ft) between sections</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>36</td>
<td>38</td>
<td>1.0</td>
<td>0.95</td>
<td>1.79</td>
<td>88</td>
<td>0.32</td>
</tr>
<tr>
<td>2.</td>
<td>38</td>
<td>34</td>
<td>1.1</td>
<td>1.10</td>
<td>1.70</td>
<td>109</td>
<td>.84</td>
</tr>
<tr>
<td>3.</td>
<td>32</td>
<td>32</td>
<td>.82</td>
<td>.86</td>
<td>1.90</td>
<td>117</td>
<td>1.28</td>
</tr>
<tr>
<td>4.</td>
<td>34</td>
<td>39</td>
<td>.9</td>
<td>.91</td>
<td>1.91</td>
<td>116</td>
<td>1.12</td>
</tr>
</tbody>
</table>

Notes.—
No. 769 downstream from section 3, Provo River near Hailstone, Utah.

No. 770 upstream from section 3, Provo River near Hailstone, Utah.
Channel $n$ Values

The most important factors that affect the selection of channel $n$ values are:

1. the type and size of the materials that compose the bed and banks of the channel
2. the shape of the channel.

Cowan (1956) developed a procedure for estimating the effects of these factors to determine the value of $n$ for a channel. The value of $n$ may be computed by

$$n = (n_b + n_1 + n_2 + n_3 + n_4)m \quad (3)$$

where:

$n_b$ = a base value of $n$ for a straight, uniform, smooth channel in natural materials

$n_1$ = a correction factor for the effect of surface irregularities

$n_2$ = a value for variations in shape and size of the channel cross section,

$n_3$ = a value for obstructions

$n_4$ = a value for vegetation and flow conditions

$m$ = a correction factor for meandering of the channel
Combining the tools

Relating $Q$ to $\tau$ in a simple, prismatic, wide channel

This is how we link transport rate (a function of $\tau$) to water discharge $Q$

Discharge = width * depth * velocity

Width increases with discharge

The ‘depth-slope’ product

Manning’s eqn

Combine. It’s only algebra!

Boundary stress is related to discharge, slope, roughness, channel shape

This solves the FLOW problem
Why?

Because we generally relate changes in the bed and banks – *erosion and deposition* – to the bed shear stress \( \tau_0 \) and we will want to be able to relate \( \tau_0 \) to the water discharge e.g. at what \( Q \) will \( \tau_0 \) be sufficient to move river bed material?

\[
Q = BhU
\]
\[
B = \alpha Q^\beta
\]
\[
h = \frac{\tau}{\rho g S}
\]
\[
U = \frac{\sqrt{S}}{n} h^{2/3}
\]
so
\[
Q = \left( \alpha Q^\beta \right) \left( \frac{\tau}{\rho g S} \right)^{5/3} \frac{\sqrt{S}}{n}
\]
or
\[
Q = \left[ \frac{\alpha \sqrt{S}}{n} \left( \frac{\tau}{\rho g S} \right)^{5/3} \right]^{1\over 1-\beta}
\]
\[
Q_c = \left[ \frac{\alpha \sqrt{S}}{n} \left( \frac{\tau_c}{\rho g S} \right)^{5/3} \right]^{1\over 1-\beta}
\]

For example, if the grain size of the river bed is used to find the critical shear stress for incipient motion \( \tau_c \), this relation gives us the critical discharge \( Q_c \) needed to move the river bed.
Conservation of Energy

- Units: Length
- Head ‘loss’
- H, EGL
- Components

\[ z_1 + h_1 + \frac{U_1^2}{2g} = z_2 + h_2 + \frac{U_2^2}{2g} + h_f \]

or

\[ H_1 = H_2 + h_f \]
• **Gradually varied flow**
  Estimate $h_f$, use energy eqn. to estimate depth at one place given depth at another place (next)

• **Rapidly varied flow**
  (gates, weirs, rapid changes in width or bottom elevation)
  flow often complex, but can be approximated by assuming negligible energy loss (*if flow converging*)

• Over short distances, with small energy loss,

\[
E = h + \frac{U^2}{2g}
\]

$z_1 \approx z_2$ and $h_f \approx 0$, so

\[
z_1 + h_1 + \frac{U_1^2}{2g} = z_2 + h_2 + \frac{U_2^2}{2g} + h_f
\]

becomes

\[
h_1 + \frac{U_1^2}{2g} = h_2 + \frac{U_2^2}{2g}
\]

or

\[
E_1 = E_2
\]
The Specific Energy $E$

flow energy relative to the bed

$E = h + \frac{U^2}{2g}$

$Q = UA$  For a rect. channel

$Q = Ubh$  Define

$q \equiv \frac{Q}{b} = Uh$

so $U = \frac{q}{h}$ and

$E = h + \frac{q^2}{2gh^2}$
The extraordinary properties of the Specific Energy $E$

- 2 flow states
- Change in $h$ with step or constriction
- Properties of Min($E$) $F = 1$

$$F = \frac{U}{\sqrt{gh}} = \frac{q}{\sqrt{gh^3}} = 1$$

To find the minimum $E$, set $\frac{dE}{dh} = 0$.

$$\frac{d}{dh} \left( h + \frac{q^2}{2gh^2} \right) = 0$$

$$1 - \frac{q^2}{gh^3} = 0$$

$$\frac{q^2}{gh^3} = \frac{U^2}{gh} = 1 = F^2$$

so $F = 1$ at minimum $E$.

Note, too, that $q = \sqrt{gh^3}$

where a subscript 'c' is included to indicate that this holds at critical flow.

$$E = h + \frac{q^2}{2gh^2}$$
What happens when flow goes over a smooth step of height $\Delta z$, or through a contraction in width, with negligible head loss ($H$ const)?

Flow Over Step

$z_1 + \Delta z = z_2$ and $h_f \approx 0$,

$z_1 + E_1 = z_2 + E_2$

or $E_1 - \Delta z = E_2$

Flow Through Contraction $E_1 = E_2$

$E = h + \frac{q^2}{2gh^2}$

$F = \frac{U}{\sqrt{gh}} = 1$

Diagrams from Roberson & Crowe, Fluid Mechanics, Wiley

Diagrams use $y$ for flow depth and $V$ for velocity
Gradually Varied Flow: Modeling Water Surface Profiles
aka backwater or step-backwater modeling

Switching back depth from $y$ to $h$ and mean velocity from $V$ to $U$ .... Sorry!

Conservation of energy in open channel flow

\[
E_1 + \frac{U_1^2}{2g} + h_1 = E_2 + \frac{U_2^2}{2g} + h_f + h_2
\]

Long distances: $Z_1 > Z_2$ and $h_f > 0$
The rate of head loss is defined as $S_f$

Must estimate head loss, so we need

length of stream over which the head loss is occurring

$\Delta x = (x_2 - x_1)$

$$z_1 + h_1 + \frac{U_1^2}{2g} = z_2 + h_2 + \frac{U_2^2}{2g} + h_f$$

$$\left( z_2 + h_2 + \frac{U_2^2}{2g} \right) - \left( z_1 + h_1 + \frac{U_1^2}{2g} \right) = -h_f = -S_f \Delta x$$

$$\frac{\left( z_2 + h_2 + \frac{U_2^2}{2g} \right) - \left( z_1 + h_1 + \frac{U_1^2}{2g} \right)}{x_2 - x_1} = \frac{\Delta H}{\Delta x} \equiv -S_f$$

Just restating the definition of energy in open channel flow
Using \[ \frac{z_2 - z_1}{x_2 - x_1} \equiv -S_o \]

\[
\left( \frac{z_2 + h_2 + \frac{U^2_2}{2g}}{x_2 - x_1} \right) - \left( \frac{z_1 + h_1 + \frac{U^2_1}{2g}}{x_2 - x_1} \right) = \frac{\Delta H}{\Delta x} \equiv -S_f
\]

becomes

\[
\left( \frac{h_2 + \frac{U^2_2}{2g}}{x_2 - x_1} \right) - \left( \frac{h_1 + \frac{U^2_1}{2g}}{x_2 - x_1} \right) = S_o - S_f
\]

An ordinary differential equation that tells us the rate \((\Delta h/\Delta x)\) at which \(h\) changes along stream. This relation only helps if we know the depth at some place to start with. If we do, then the equation can be used to calculate depth at some other location and we are off to the races.
Given $h_1$ at $x_1$, at what $x_2$ will $h_2$ occur?

**Direct Step**

Specify $Q$, $S_o$, and $h_1$.
Continuity gives $U_1$.
Pick $h_2$, continuity gives $U_2$.
Solve for $x_2$.

\[
\frac{\left( h_2 + \frac{U_2^2}{2g} \right) - \left( h_1 + \frac{U_1^2}{2g} \right)}{x_2 - x_1} = S_o - S_f
\]

\[
x_2 = x_1 + \frac{\left( h_2 + \frac{U_2^2}{2g} \right) - \left( h_1 + \frac{U_1^2}{2g} \right)}{S_o - S_f}
\]

Not very convenient, except for prismatic canals!
Given $h_1$ at $x_1$, what is $h_2$ at $x_2$?

Specify $Q$, $S_o$, $h_1$ and $\Delta x$

Continuity gives $U_1$

**Guess** $h_2$, continuity gives $U_2$

Iterate.
Defining $S_f$. We used Manning’s eqn to describe flow resistance

For steady, uniform flow, $S_o = S_f$ and we did not have to specify which $S$ was used in Manning’s eq.

We now precisely define the slope in Mannings’s eqn as $S_f$ and we use Manning’s eq. to define $S_f$.

$$h_2 = h_1 + \frac{1}{2g} \left( U_1^2 - U_2^2 \right) (S_o - S_f) (x_2 - x_1)$$

Having chosen a value of $h_2$, we calculate $R_2$, $U_2$, and $S_f$. Inserting $U_2$ and $S_f$ on the right side, we then calculate a new, improved guess for $h_2$.

This is step-backwater modeling. This is HEC-RAS.
Big advantages of a canned program:

(a) Iterative solution to the governing equation is not always stable

(b) The world is messy (irregular cross-sections, variable roughness, compound channels, bridges, culverts, ....)

Don’t believe a RAS solution unless you can sketch it yourself! How?

(1) identify controls (known depth for a specified discharge)
   critical depth, normal flow

(2) which way to go (start D/S in subcritical flow and U/S in supercritical flow

(3) Sketch!

In its most basic application, to build a HEC-RAS model you (a) survey cross sections in your study reach and enter these into RAS, (b) assign roughness to each section (or portions of sections), (c) specify a discharge, and (d) step back and let it run.

ROUGHNESS: If you don’t calibrate, you are only guessing.
Types of gradually varied flow profiles

Subcritical Flow – Downstream Control!

Supercritical Flow – Upstream Control!

\[ h_c = \left( \frac{q^2}{g} \right)^{1/3} \]

aka \( F = 1 \)

\[ h_n = \left( \frac{nq}{\sqrt{S}} \right)^{3/5} \]

aka Manning’s eq.
Some Key Points

<table>
<thead>
<tr>
<th>How often does sediment move?</th>
<th>What is the Sediment Balance?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow <strong>COMPETENCE</strong> of bed material</td>
<td>Sediment Supply v. Transport <strong>CAPACITY</strong></td>
</tr>
<tr>
<td>Threshold Channel, Incipient motion</td>
<td>Alluvial channel problem</td>
</tr>
<tr>
<td>Shields curve + flood frequency</td>
<td>Transport relations give capacity. Supply?</td>
</tr>
</tbody>
</table>

All sediment transport problems have a FLOW PART
- **Flow problem** → hydraulics → boundary stress $\tau_o$
- **Sediment problem** → sediment response to boundary stress $\tau_o$

All relations are approximate. You can quantitatively evaluate the error associated with approximations (e.g. $\tau_o = \rho g RS$)

Channel hydraulics and transport: uncertainty is in the input!
- can estimate the magnitude and importance of uncertainty
- need actual measurements to obtain accuracy

The tools are general, their application is local.

**RAS** *(et al.)* calculates sediment transport? **Plug and Play?**
- Provides useful basis for estimating flow and stress
- Accounts (coarsely) for nonuniform flow conditions

**RAS is very good at what it is meant to do: calculate water surface elevation.**
**For sediment transport: it’s so easy to be so wrong**
(1a) Finding Normal Depth - Rectangular Channel

V = \( \frac{R_h^{2/3}}{n} \sqrt{S_o} \) and \( Q = \frac{A^{5/3}}{nP^{2/3}} \sqrt{S_o} \)

where

\[ A = bh, \quad P = b + 2h, \quad R_h = \frac{bh}{b + 2h} \]

Given \( R_h, n, Q \) ⇒ Find \( S_o \) from ⊕

Given \( R_h, n, S_o \) ⇒ Find \( Q \) from ⊕

Given \( R_h, Q, S_o \) ⇒ Find \( n \) from ⊕

Given \( S_o, n, Q \) ⇒ To find \( h \), have to iterate!

Try ⊕ arranged as \( Q = \frac{b}{n} \frac{h^{5/3}}{(1 + \frac{h}{b})^{2/3}} \sqrt{S_o} \)

Solve for \( y \) on top: \( h_{n+1} = \left( \frac{nQ}{b\sqrt{S_o}} \right)^{3/5} \left( 1 + \frac{h_n}{b} \right)^{2/5} \)

where \( n \) and \( n + 1 \) subscripts indicate successive approximations
(1b) Finding normal depth in a trapezoidal channel

Manning's eqn:  
\[ Q = \frac{\sqrt{S_o}}{n} \left[ \frac{h(B_o + sh)}{B_o + 2h\sqrt{1 + s^2}} \right]^{2/3} \]

where

\[ B = B_o + 2sh \quad A = B_o h + sh^2 = h(B_o + sh) \quad P = B_o + 2h\sqrt{1 + s^2} \]

Given \( h,n,Q \) \Rightarrow Find \( S_o \) from

Given \( h,n,S_o \) \Rightarrow Find \( Q \) from

Given \( h,S_o,Q \) \Rightarrow Find \( n \) from

Given \( S_o,n,Q \) \Rightarrow To find \( h \), have to iterate!

Try \( \otimes \) arranged as \( h_{n+1} = \left( \frac{nQ}{\sqrt{S_o}} \right)^{3/5} \left[ \frac{B_o + 2h_n\sqrt{1 + s^2}}{B_o + sh_n} \right]^{2/5} \)

where \( n \) and \( n + 1 \) subscripts indicate successive approximations.
(2) Finding critical depth in a rectangular or trapezoidal channel

(2) Finding critical depth

Finding Critical Depth \( F = 1 = \frac{V}{\sqrt{gh}} = \frac{Q/A}{\sqrt{g(A/B)}} \) or \( \frac{Q}{\sqrt{g}} = \frac{A^{3/2}}{\sqrt{B}} \)

where \( B \) is top width.

(2a) Rectangular Channel \( \frac{Q}{\sqrt{g}} = \frac{A^{3/2}}{\sqrt{B}} = \frac{(Bh)^{3/2}}{\sqrt{B}} \) or \( \bar{h}_c = \left( \frac{Q}{\sqrt{g}B} \right)^{2/3} \)

(2b) Trapezoidal Channel

\( B = B_o + 2sh \quad A = B_o h + sh^2 = h(B_o + sh) \)

\( \frac{A^{3/2}}{\sqrt{B}} = \sqrt{\frac{h^3(B_o + sh)^3}{B_o + 2sh}} \)

\( \frac{Q}{\sqrt{g}} = \sqrt{\frac{y^3(B_o + sh)^3}{B_o + 2sh}} \)

Given \( h_c, B_o \) \( \Rightarrow \) Find \( Q = \sqrt{\frac{gh_c^3(B_o + sh_c)^3}{B_o + 2sh_c}} \)

Given \( Q, B_o \) \( \Rightarrow \) Find \( y_c = \left( \frac{Q^2}{g} \frac{B_o + 2sh_c}{(B_o + sh_c)^3} \right)^{1/3} \)

Given \( h_c, Q \) \( \Rightarrow \) Find \( B_o = \frac{gh_c^3}{Q^2}(B_o + sh_c)^3 - 2sh_c \)
When do I use momentum and when do I use energy?

They are both always true, but in most cases, one term in one of the two conservation relations is poorly known and, therefore, you don’t have the information needed to solve the problem. Often, you can use the other relation to solve the problem and then determine the unknown forces (momentum) or head losses (energy).

Expanding Flow: head losses are likely to be nonnegligible, whereas the flow force on the wall may be small. Thus, you use momentum assuming no wall force. After solving for the velocity in both sections, you can use energy to find the head loss.

Contracting Flow: flow force on the wall is likely to be nonnegligible, whereas the head losses may be small. Thus, you use energy assuming no head loss. After solving for the velocity in both sections, you can use momentum to find the wall force.

For standard channels and pipes, head loss has been carefully measured experimentally and head-loss coefficients are available to directly solve the energy problem.
Some comments on using a 1d flow model to estimate sediment transport and channel change
Comments on sediment transport model zeroing, with reference to Marmot Dam Model

The objective of a zeroing process is to adjust the model input parameters so that they approximately reproduce the existing quasi-equilibrium long profile under the assumed background conditions. One assumption, therefore, is that the modeled reach is in a quasi-equilibrium condition, in which some aggradation or degradation may occur following flood events, but the long-term cumulative channel aggradation or degradation is minimal. The zeroing process involves running the model repeatedly with the surveyed longitudinal profile as the initial condition and the recorded hydrologic condition and best estimate of sediment supply rate and grain size distribution as boundary conditions. During this process, certain input data such as channel width, sediment supply rate, and/or grain size distribution are adjusted iteratively until the model reproduces a quasi-equilibrium profile similar to that observed. The reproduced quasi-equilibrium longitudinal profile is then used as the initial profile for modeling future conditions such as evaluating sediment transport dynamics following dam removal herein or following channel reconstruction for restoration. Because this initial profile is in quasi-equilibrium state within the model, any deviations from this condition in the subsequent simulations are considered to result from the perturbation injected into the model input.

During the zeroing process for Marmot Dam removal sediment transport study, the following adjustments were made: channel width was adjusted by narrowing some of the excessively wide cross sections, long-term averaged sediment supply rate was adjusted within the range found during the literature review, and the abrasion coefficient of gravel particles was also adjusted based on published range so that the predicted grain size distribution and longitudinal profile under the current conditions were similar to observations.

Comments on multidimensional modeling, with reference to Marmot Dam Model

There are three main limitations for multidimensional numerical models. First, while multidimensional modeling can usually realistically reproduce the flow field, the detailed relation between sediment transport and movement of sediment particles is not fully understood, and all the current sediment transport equations were developed based on data collected on a cross-section averaged basis. As such, topography predicted as a direct result of flow field (such as scour due to river bend) can often be realistically modeled, but the topography associated with complex sediment transport dynamics, such as the formation and development of alternative bars in a straight channel, may not be realistically reproduced. Second, attempting to model sediment transport dynamics in detail requires the collection of detailed field data, some of which are critical to the modeling but impractical to obtain in many situations or at a large scale. For example, simulating detailed topography in an area subject to channel erosion will require the knowledge of detailed grain size distributions and information with regard to where nonerodible material (such as bedrock and large boulders) is located and how deep it is beneath the surface. While it is possible to make some generalized assumptions about grain size distributions based on observations of the surface or bulk samples, it is impractical to know the locations and depth of the bedrock and large boulders, and without such information, the modeling results with regard to future topography would not have the desired resolution. Third, limitations in available computer resources will set upper bounds on the number of nodes permissible in a multidimensional model simulation, and because computational meshes cannot be overly distorted (i.e., the longitudinal dimension of the meshes cannot be too much larger than the lateral dimension), there are practical limits on the length of the river that can be simulated. This latter issue can introduce a further problem in that modeling short reaches may make the entire simulation domain dependent on boundary (especially downstream boundary) conditions, making the simulation results unreliable. This is generally not an issue in 1-D modeling because a 1-D model can be set to a significantly longer reach so that the interested area is beyond the influence of the model boundary.

The primary reason that multidimensional numerical modeling was not proposed at the time was that 1-D numerical modeling had satisfactorily answered all the important questions that the stakeholders and regulating agencies needed to know, with a few uncertainties addressed with contingency plans discussed earlier, allowing the stakeholders to reach an agreement. In addition, multidimensional simulations would have been limited to only a short period of time following dam removal, which was not the primary interest of the stakeholders.

Flow and Transport Modeling to Support Decision Making in the Management of Glen Canyon Dam

Peter Wilcock
Johns Hopkins University

Stephen Wiele, Scott Wright
USGS

2005 SCORE Report
USGS Circular 1282
The management issue

- Can sand bars be restored & maintained by modified dam operations (alone)?
  - high flow releases used to build bars, storing sand at high elevation
  - low flows used to conserve tributary sand input until high flow release is possible

- Need simple operational guidelines & testable hypotheses
The science challenge: predict transport and storage of sand as fn(tributary inputs, discharge)

87 miles of complex channel geometry

Limited access

Most of the sand in eddies with erosion and deposition in recirculating flow

Options:
1d hydraulic routing model
2d or 3d morphodynamic model
1d reach-averaged routing model with coupled sand storage fns
Key Model Elements

1) Reach-averaged channel geometry
   (complex channel shape, incomplete bathymetry)
   6 reaches: Paria R Confluence to Phantom Ranch (87 miles)
   pipe + pools

2) 1d unsteady flow model (significant daily fluctuations)
   existing reach-averaged flow model

3) Near-bed sand concentration
   (suspended sand transport over a cobble/boulder bed)
   develop independent relation for sand entrainment

4) Sand storage in eddies (cannot be captured in a 1d model)
   source/sink fns derived from 2d model applied to
   7 km-scale reaches @ 10 discharges & 3 sand concentrations

1. Reach-averaged channel geometry
1. Reach-averaged channel geometry
   - Cross sections (Griffin, 1997)

   - Stage-discharge (Wiele and Torizzo, 2003)
1. Reach-averaged channel geometry – Pools (Leopold, 1965, Randle and Pemberton, 1984)
2. 1d unsteady flow model

-- reach-averaged hydraulic geometry from

• streamflow gage records
• kinematic wave speed --

integrate function for wave speed
to get $Q = f(A)$

• dye study (Graf) for constant of Integration

Wiele and Smith, 1996
Wiele and Griffin, 1997
3. Near-bed sand concentration over a cobble/boulder bed
Sand Entrainment Function

- **Sand Entrainment Rate**
- **Entrainment Sand Bed**

(0,0) No Sand → no sand entrainment

(1,1) Sand bed ... sand-bed entrainment rates

Sand Bed Elevation
Roughness Height
Main channel bed – Run 4

Run 4A 91 min.

Run 4B 144 min.

Run 4C 229 min.

Run 4D 348 min.

40 m
1 + 2 + 3 → Sand routing

• Calculated at local node (about ½-km apart) based on discharge, sand supply, and reach hydraulic properties
• Skin friction from Einstein decomposition
• Near bed sand concentration from Grams-Wilcock modification to Garcia-Parker relation
• Sand concentration from Rouse profile

Eddy erosion and deposition are invisible to this part of the model ...
4. Sand erosion & deposition in eddies

2d model of flow, sand transport, and bed evolution applied to 7 km-scale reaches @ 10 discharges & 3 sand concentrations to develop source/sink functions
4. Sand erosion & deposition in eddies

calculate local flow & transport fields as function of discharge, sand supply, & initial sand volume
→ predict local sand deposition and scour
4. Dimensionless relations for sand deposition & scour

**Eddy deposition**
modeled as function of
• eddy properties
• discharge
• sand concentration
• time \((dv/dt)\)

**Eddy scour**
modeled as a function of
• eddy properties
• discharge
• change in discharge \((dv/dq)\)

For each time step in each model reach:
use summary eddy geometry to calculate sand additions and subtractions
A few Lessons Learned from Sediment Transport Modeling Projects

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Stillwater Sciences (www.stillwatersci.com)

Presented at 2011-05-25 ASCE EWRI Conference, Palm Springs

And

Some comments on flushing flows
Ecological Application: impact of bed material fines on salmonid eggs & emerging fry, and juvenile growth & mortality
Effect of fine (< 2 mm) sediments on juvenile steelhead and the food webs that support them (Suttle, Power, Levine, McNeely, Sapp, Sorenson)

6 levels: 0, 20, 40, 60, 80 and 100% embedded

2 m x 0.5 m flow through channels

Replicated at 4 sites over ca. 4 km of S. Fk. Eel

8/2/2022
Growth in length and mass decreased linearly with proportion of fine sediment.
This occurred in part, because benthic invertebrate assemblages in embedded treatments were made up of less available and nutritious taxa.
...and in part because fish were more active (and aggressive towards each other) on the flat, featureless beds in embedded treatments,

Mortality increased with embeddedness, and was associated with wounds from fighting.
Ecological Application: reducing fines concentration in gravel-bed rivers

Solution: sufficient flow to entrain most of the bed surface (to flush the subsurface) and reduction of sand supply to below sand transport capacity

Note: both the competence and capacity elements of sediment transport are invoked here!

**Flow Competence**: flow must be high enough and last long enough to move most of the coarse grains on the bed surface

**Transport Capacity**: the reach must transport fines faster than the rate at which they are supplied
A flow of 6,000 cfs was sufficient to mobilize most of the gravel. Food for flushing IF the sand content of Trinity River can be reduced.